MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2016/2017

TMA1101 – CALCULUS

(All sections / Groups)

22 OCTOBER 2016
9:00 a.m. – 11:00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of six pages with **FIVE** questions.
2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the answer booklet provided.
4. **No calculators are allowed**.
5. **You are required to write proper steps.**

ANSWER ALL QUESTIONS.

QUESTION 1 [10 marks]

1 (a) Find the following limits.

[***You must show at least at least one intermediate step where*** $\lim_{x\to c}$ ***is still needed.***]

(i) $\lim_{x\to 4}\dfrac{x^2-x-12}{x^2-16}$

(ii) $\lim_{x\to 0}\dfrac{(x+2)^2-4}{x}$ [3 marks]

(b) Given $f(x)=\begin{cases} x+5, & \text{if } x<4 \\ 2x, & \text{if } x=4 \\ x^2-7, & \text{if } x>4 \end{cases}$

(i) Find $\lim_{x\to 4^-} f(x)$ and $\lim_{x\to 4^+} f(x)$.

[***For this part, you must show at least at least one intermediate step where*** $\lim_{x\to 2^-}$ ***or*** $\lim_{x\to 2^+}$ ***is still needed.***]

(ii) Does $\lim_{x\to 4} f(x)$ exist? Give your reason. If it exists, state its value.

(iii) Determine whether f is continuous at 4. Give your reason.

[4 marks]

(c) (i) State the Intermediate Value Theorem (i.e., the full statement including the hypothesis and the conclusion).

(ii) Show that there is a root of the equation $2x^3-6x+1=0$ in the interval $[1, 2]$.
You must write proper steps to arrive at the conclusion; just writing some calculations would not be enough.

[3 marks]

Continued .....

QUESTION 2 [10 marks]

(a) Use the formal definition of derivative to find $f'(2)$ given $f(x) = x^2 - 2x$.
You are reminded to write proper steps. [2.5 marks]

(b) Find the derivative of y with respect to x.

$$y = \frac{x}{1 + \cos x}$$ [1 mark]

(c) Show that the function $f(x) = x \sin x$ has a local minimum at $x = 0$.

[2.5 marks].

(d) The point (1, 2) lies on the curve $y^2 + 3xy + x^2 = 11$.

Use implicit differentiation to obtain $\frac{dy}{dx}$ in terms of x and y.

Hence, find the equation of the tangent line to the curve $y^2 + 3xy + x^2 = 11$ at the point (1, 2).

[4 marks]

Continued .....

QUESTION 3 [10 marks]

(a) (i) Determine the values of A and B in the following partial fraction decomposition.

$$\frac{3x+7}{x^2-2x-3} = \frac{A}{x-3} + \frac{B}{x+1}$$

(ii) Then evaluate $\int \frac{3x+7}{x^2-2x-3} dx$ [3 marks]

(b) (i) Use $\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$ to find the values of A and B which make the equation $\cos^3 x = A\cos 3x + B\cos x$ an identity.

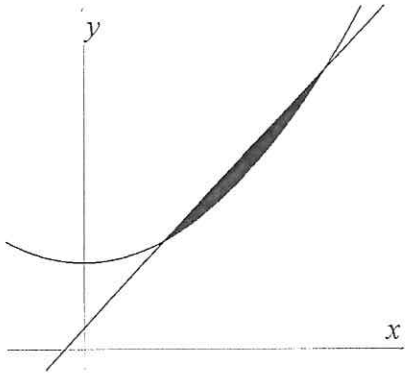
[The identity $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ may be useful.]

(ii) Evaluate $\int_0^{\frac{\pi}{4}} \cos^3 x \, dx$ [4 marks]

(c) (i)

The figure shows a region bounded by the parabola $y = x^2 + 4$ and the straight line $y = 4x + 1$.

Determine the x-coordinates of the points of intersection between the parabola and the straight line.



(ii) Write down a definite integral that can be used to find the area of this region and proceed to find the area.

[3 marks]

Continued .....

QUESTION 4 [10 marks]

(a) Given the sequence $\{a_n\}$ with $a_n = \dfrac{3n^2 - 1}{10n + 5n^2}$.

Determine $\lim_{n\to\infty} a_n$; you are reminded to write proper steps.

Then determine whether the infinite series $\sum_{n=1}^{\infty} \dfrac{3n^2 - 1}{10n + 5n^2}$ is convergent.

Give the reason for your answer.

[2.5 marks]

(b) Let $a_n = \dfrac{2^n}{n^2}$. Determine $\lim_{n\to\infty} \dfrac{a_{n+1}}{a_n}$.

Then use the **ratio test** to determine whether the infinite series $\sum_{n=1}^{\infty} \dfrac{2^n}{n^2}$ is convergent.

[2.5 marks]

(c) Find the Maclaurin polynomial of order 3 for $f(x) = \dfrac{1}{(1-x)^2}$. [3 marks]

(d) A periodic function f of period 2π is defined as

$$f(x) = \begin{cases} 0 & -\pi \le x < 0 \\ \pi & 0 \le x < \pi \end{cases}$$

The **Fourier series** of f has the form $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$.

Find b_3.

[2 marks]

Continued .....

QUESTION 5 [10 marks]

(a) Find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the function

$$f(x, y) = x^2 - xy\sin x.$$ [1 mark]

(b) Solve the first order **separable equation** $\frac{dy}{dx} = \frac{1}{x^2 y}$

subject to the initial condition $y(1) = 3$.

You may leave your answer in implicit form. [2.5 marks]

(c) You are told that e^{2x} is an integrating factor for the first order linear equation

$$\frac{dy}{dx} + 2y = e^x$$

Solve the equation and give your solution in explicit form. [2.5 marks]

(d) (i) Find the roots of the **characteristic equation** of the homogeneous differential equation $y''-4y'-12y = 0$.

Then write down the general solution y_h of this homogeneous differential equation.

(ii) If $y = Ae^{5x}$ is a **particular solution** of the differential equation $y''-4y'-12y = 3e^{5x}$, determine the value of A.

(iii) Hence, write down the **general solution** for the differential equation $y''-4y'-12y = 3e^{5x}$. [4 marks]

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